The Modulus of Rigidity, $\eta$, also known as the Shear Modulus, is a property of materials which denotes how the material responds to a Shear Force. It describes how much a material deforms when a force is applied perpendicular to its axis. It is a property which is of critical importance to structural engineers since it determines how a structure will respond to lateral forces (wind, earthquakes, etc.).

Spring Constant

We have already studied Hooke’s Law, $F = kx$, which governs the behaviour of a spring. The spring constant $k$ depends upon the shape of the spring (its geometry), the thickness of the wire, and the Modulus of Rigidity of the material used to construct it.

$$ k = \frac{\pi r^4 \eta}{2lR^2} $$  \hspace{1cm} (1)

where $R$ is the radius of the spring, $r$ is the radius of the wire and $l$ is the length of the wire that makes up the spring. If the spring has $N$ turns and a radius of $R$ then $l = N2\pi R$ and so

$$ k = \frac{\eta r^4}{4NR^3} $$  \hspace{1cm} (2)

Time Period

Equation (2) means we can measure the modulus of rigidity of a material (such as steel) by using a spring constructed from that material. If a mass $M$ is hung from the spring and allowed to oscillate then the time period of the oscillation is given by

$$ T = 2\pi \sqrt{\frac{M}{k}} $$  \hspace{1cm} (3)

Substituting in equation (2) gives us

$$ T = \frac{4\pi}{r^2} \sqrt{\frac{NR^3 M}{\eta}} $$  \hspace{1cm} (4)
Experiment 5: Modulus of Rigidity

Objective

Calculate the modulus of rigidity, $\eta$, of steel using a spring.

Concept

The time period of a spring-mass system in terms of the geometry of the spring and the modulus of rigidity of its material is given by:

$$T = \frac{4\pi}{r^2} \sqrt{\frac{NR^3M}{\eta}}$$

where $R$ is the radius of the spring, $N$ is the number of turns of the spring, $r$ is the radius of the wire that makes up the spring, and $M$ is the mass suspended from the spring.

Design

Use equation (1) to design an experiment that will allow you to calculate the modulus of rigidity $\eta$. Answer the following questions to guide your design.

(i) What is the independent variable in this equation? That is, what is the quantity you will vary?

(ii) What is the dependent variable? That is, what will you measure every time you change the quantity above?

(iii) What are the constants? The quantities you will keep fixed and measure only once.

(iv) How will you measure each of these quantities? This determines your choice of apparatus.

(v) What will you plot on your graph?

(vi) How will you determine the uncertainty in your measurements?

Requirements

You are required to

(i) Design the Experiment.

(ii) Provide a List of Apparatus

(iii) Write the Procedure

(iv) Take measurements and record them in a Table
(v) Draw a graph
(vi) Calculate the value of $\eta$ along with its uncertainty.