COMSATS Institute of Information Technology,
Islamabad Campus

PHYS-108

Experiment 6: Viscosity (Stoke's Law)

Viscosity is a property of fluids (liquids and gases) which determines how much resistance is experienced by an object trying to move through the fluid. In this experiment we will use Stoke's Law and the concept of terminal velocity to determine the viscosity of glycerin.

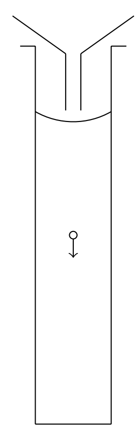
Objective

Use Stoke's Law to derive an equation relating the viscosity η of a fluid to the time t it takes for a sphere to fall a distance d through it.

Experimental Setup

The equipment consists of a long glass cylinder filled with glycerin. A graduated scale is attached to the cylinder which allows us to measure distances along the cylinder.

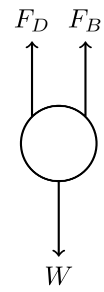
Small metallic spheres are dropped from the top of the cylinder. The viscosity of the glycerin produces a drag force that will eventually result in terminal velocity being achieved by the falling sphere. The terminal velocity is calculated by measuring the time t it takes the sphere to fall through a distance d inside the fluid.



Free-Body Diagram

A sphere falling through a fluid experiences three forces: its weight W, an upward buoyant force F_B (Archimedes' Principle) and the drag force F_D (Stoke's Law) also in the upward direction since it opposes the downward motion of the sphere.

 Edited by Abid H. Mujtaba (2016)



Derivation

The three forces acting on the falling sphere are given by

$$\begin{aligned} W &= mg \\ F_B &= \sigma\, V g \\ F_D &= 6\pi\,\eta\, r v \end{aligned} \tag{1}$$

where σ is the density of the fluid and the last equation is a statement of Stoke's Law which describes the drag force acting on a sphere of radius r as it moves with velocity v through a fluid with viscosity η.

The first two forces remain constant but the drag force increases in magnitude as the sphere speeds up since it is directly proportional to the velocity v. Initially, when the velocity of the sphere is low, the drag force is low as well and therefore the net downward force and acceleration are large. This causes the velocity to increase. As the velocity increases the drag force increases and in turn the net downward force and acceleration decrease. The velocity keeps increasing but at a slower rate. Eventually the drag force increases until it exactly balances the other two forces. The net downward force and acceleration become zero and the velocity becomes constant. This is known as the terminal velocity v_T.

Therefore, by definition, when the terminal velocity is achieved the net force on the sphere is zero, the three forces balance each other out. Looking at the free-body diagram this means

$$W = F_B + F_D \tag{2}$$

Substituting (1)

$$\Rightarrow\ mg = \sigma\, V g + 6\pi\,\eta\, r v_T$$

The mass of the sphere m can be expressed in terms of the density ρ of the sphere as $m = \rho V$.

$$\begin{aligned} &\Rightarrow\ \rho V g = \sigma\, V g + 6\pi\,\eta\, r v_T \\ &\Rightarrow\ 6\pi\,\eta\, r v_T = \rho V g - \sigma V g \\ &\Rightarrow\ 6\pi\,\eta\, r v_T = (\rho - \sigma) V g \end{aligned}$$

The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$. If the sphere (after attaining terminal velocity) falls through a distance d in time t the terminal velocity can be calculated using $v_T = \frac{d}{t}$. Substituting these in

$$\begin{aligned} &\Rightarrow\ 6\pi\,\eta\, r\frac{d}{t} = (\rho - \sigma)\frac{4}{3}\pi r^3 g \\ &\Rightarrow\ 3\,\eta\,\frac{d}{t} = \frac{2}{3}(\rho - \sigma) g\, r^2 \end{aligned}$$

Since our aim is to calculate the viscosity of the fluid we make η the subject of the equation giving us

$$\eta = \frac{2}{9}\frac{(\rho - \sigma) g\, r^2 t}{d} \tag{3}$$

Thus we end up with an equation that relates the viscosity of a fluid to the radius of the sphere falling through it and the time it takes for it to fall through a certain distance. This equation governs the working of a **viscometer** a device where falling spheres are used to measure the viscosity of a fluid.

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Objective

Calculate the viscosity η of glycerin.

Apparatus

- Graduated glass cylinder (50 cm long)
- Steel ball-bearings
- Conical Flask
- Stopwatch
- Micrometer
- Digital Balance
- Magnet attached to a string

Procedure

The viscosity of a liquid can be measured using a viscometer. A viscometer consists of a graduated glass cylinder filled with the liquid in question, in our case glycerin. Small metallic spheres are dropped in to the liquid from the top. After falling sufficiently the spheres acquire terminal velocity. The viscosity of glycerin is given by

$$\eta = \frac{2}{9} \frac{(\rho - \sigma) g \, r^2 t}{d} \tag{1}$$

where

- η is the viscosity of glycerin
- ρ is the density of the sphere
- σ is the density of glycerin
- g is the acceleration due to gravity
- r is the radius of the sphere
- d is the distance between two markers (AB = BC = d)
- t is the time it takes for the sphere to cover the distance d

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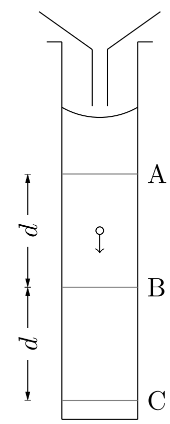
Setup

The cylinder is filled with glycerin. The magnet tied to a string is gently lowered to the bottom of the cylinder such that a portion of the string hangs over the top edge. The purpose of the magnet is to aid in retrieving the metallic spheres we drop in to the glycerin. All of the spheres will fall to the bottom and become attached to the magnet. When one wants to retrieve the spheres simply pull on the string to raise the magnet and the attached spheres. *Use tissue paper to handle the string so as to avoid getting glycerin on your hands or clothes. Don't worry, glycerin is perfectly harmless and washes out easily.*

Place the conical flask on top of the cylinder. The metallic spheres are to be dropped through the conical flask to ensure that they fall through the center of the cylinder.

Choosing Markers

The first part of the experiment is determining where terminal velocity is achieved. Three points/markers (A, B, C) need to be chosen on the cylinder such that the distance AB = BC. For example if A is at the 15 cm mark (on the scale attached to the cylinder) and B is at 30 cm then C must be placed at 45 cm so that AB = BC = d = 15 cm.



Initially choose A and B arbitrarily. Choose C to ensure that AB = BC. Now drop a sphere and measure the time t_1 and t_2 it takes for the sphere to move from A to B and B to C respectively. If terminal velocity has been achieved before the sphere reaches A we expect $t_1 = t_2$ (the two values should be within 0.5 s). If terminal velocity is not achieved before A then we will observer $t_1 > t_2$ since the velocity at A will be larger than at B.

If the two times don't match move the point A lower to allow the sphere to achieve terminal velocity. Move B and C accordingly. Repeat until the two times are approximately equal.

Tabulation

Record the position of the markers in a table with the following format. *Don't forget to write down the units.*

A ( )	B ( )	C ( )

Record the constants in a table with the following format. *Pay close attention to the units of d.*

d (cm)	σ (g/cm^3)	g (cm/s^2)
		981

Record your observations in a table with the following format.

D (cm)	m (g)	t_1 (s)	t_2 (s)

where D is the diameter of the sphere, t_1 is the time it takes for the sphere to move from A to B, and t_2 is the time it takes for it to move from B to C.

Constants

The density of the glycerin σ is written on its container (bottle). Note it down in g/cm^3. We also require the acceleration due to gravity. We will use the value $g = 981\,\mathrm{cm/s^2}$

Observation

Choose 5 spheres with different diameters. For each sphere measure its mass m using the digital balance, and its diameter D using the micrometer. Note these down in your table.

Now drop the sphere in to the glycerin using the conical flask placed on top of the cylinder. Use two stop-watches to measure the time t_1 it takes to move from A to B and time t_2 it takes to move from B to C. Note these down in the table as well.

Calculations

Use your measurements to calculate and record the derived quantities in a table with the following format

D (cm)	r (cm)	ρ (g/cm^3)	t (s)	η ( )

where

$$r = \frac{D}{2}$$

$$\rho = \frac{m}{V} = \frac{3\,m}{4\pi r^3}$$

$$t = \frac{t_1 + t_2}{2}$$

Use (1) to calculate both the values and **units** of η.

Results

Having performed the experiment using 5 spheres you should now have 5 values for the viscosity. Calculate the average and standard deviation of these values to come up with a final value for the viscosity of glycerin.