COMSATS Institute of Information Technology,
Islamabad Campus

PHYS-108

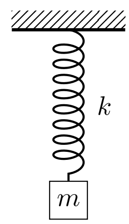
Experiment 2: Hooke's Law

Hooke's Law is a physical principle that states that a spring stretched (extended) or compressed by some distance produces a restoring force which is directly proportional to said distance. Mathematically, if an extension x is accompanied by a restoring force F then they are related by the equation

$$F = k\,x \tag{1}$$

where k is the Spring Constant.

Experimental Setup

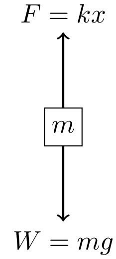


The weight of the mass attached to the bottom of the spring provides the force necessary to stretch it. The extension of the spring is measured using the pointer attached to the mass hanger. The position of the pointer is read off from the ruler placed behind the pointer.

Balancing Forces

If a weight is attached to an initially unextended spring the force applied by the weight is unbalanced and causes the spring to begin stretching (extending). As the extension of the spring increases it produces a restoring force given by (1) whose purpose is to oppose the stretching and return the spring to its initial unstretched condition. When the spring has extended enough the restoring force exactly balances the weight of the mass and the system achieves equilibrium.

This is demonstrated by a free-body diagram of the mass where the only two forces acting upon it are its weight and the restoring force exerted by the spring.



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Therefore at equilibrium (when the mass-spring system is at rest) the two forces must be balanced and so

$$\begin{aligned} F - mg &= 0 \\ \Rightarrow \; kx - mg &= 0 \\ \Rightarrow \; kx &= mg \\ \Rightarrow \; x &= \frac{g}{k}m \end{aligned} \tag{2}$$

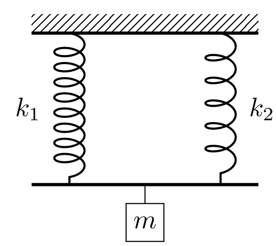
The final equation gives the relation between the mass attached to the string and the extension it produces.

1 Single Spring

Using equation (2) one can calculate the spring constant k of a spring my performing an experiment where one varies the attached mass m and measures the corresponding extension x. The slope of the graph of x vs. m is then given by g/k.

2 Springs in parallel

A mass can be suspended from two springs in parallel by connecting the bottoms of the two springs by a pole and suspending the mass from its center, as shown in the diagram below.

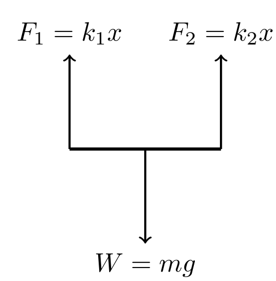


The weight of the mass causes both springs to be extended by the same amount x since they are connected by means of the pole. The two springs are essentially responding to the weight of the mass by extending and applying a restoring force. Thus together they are behaving as a single spring would behave.

The question is: What is the *effective* spring constant of the two parallel springs? That is, for what value of the spring constant would a **single** spring have the same effect as the two springs in parallel.

Let the two springs have spring constants k_1 and k_2. Let the effective spring constant (of the equivalent single spring) be k. If the application of mass m to the system produces an extension x we can use (2) to relate x to m using the spring constants. For the single spring case the equation is straight-forward, it is simply (2).

For the springs in parallel we draw a free-body diagram and equate the upward and downward forces.



Since the two springs in parallel have the same extension x we have

$$k_1x + k_2x = mg \tag{3}$$

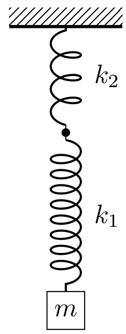
Since the parallel spring system and the effective single spring are by definition equivalent we can use (2) to substitute for mg in (3) giving us

$$\begin{aligned} kx &= mg = k_1x + k_2x \\ \Rightarrow\ kx &= (k_1 + k_2)x \\ \Rightarrow\ k &= k_1 + k_2 \end{aligned} \tag{4}$$

Therefore the *effective* spring constant of two springs in parallel is simply the sum of their individual spring constants. This basically indicates that it is **harder** to stretch two springs in parallel than it is to stretch either one by itself.

3 Springs in series

A mass can be suspended from two springs in series by connecting the bottom of upper spring to the top of the lower spring, as shown in the diagram below.



The weight of the mass causes the lower spring to extend which in turn applies a force on the upper spring causing it to extend as well. The force experienced by both springs is equal to the weight of the object. This can be demonstrated by drawing free-body diagrams (below) of the point where the mass is attached and the point where the two springs meet. At equilibrium both of these points are stationary and so the net force acting on them must be zero.

Equating the upward and downward forces gives us $F_1 = W$ and $F_2 = F_1 = W$. Therefore in the series configuration each spring experiences the same force as the lowest spring. Each spring will response to this force with a different extension x_1 and x_2 since they have different spring constants k_1 and k_2. The net extension of the two-spring system is then the sum of the extensions of each individual spring: $x = x_1 + x_2$. We can now calculate the effective spring constant k (a single spring with the same extension as the two springs in series) using (1) which tells us that $x = \dfrac{F}{k}$.

$$\begin{aligned} x &= x_1 + x_2 \\ \Rightarrow\ \frac{F}{k} &= \frac{F_1}{k_1} + \frac{F_2}{k_2} \\ \Rightarrow\ \frac{W}{k} &= \frac{W}{k_1} + \frac{W}{k_2} \\ \Rightarrow\ \frac{1}{k} &= \frac{1}{k_1} + \frac{1}{k_2} \end{aligned} \tag{5}$$

Therefore the *effective* spring constant of two springs in series is the inverse of the sum of the inverse of their individual spring constants. This basically indicates that it is **easier** to stretch two springs in series than it is to stretch either one by itself.

4 Oscillating Spring

When a spring is deformed (either stretched or compressed) a restoring force is set up which wants to bring the spring back to its original position. In a spring-mass system this means works must always be done on the mass to move it away from the mean/equilibrium position. Equivalently a mass gains Elastic Potential Energy whenever it is displaced from the equilibrium position.

If we stretch a spring-mass system and release it from rest we basically store Elastic Potential Energy inside it. After being released the restoring force pulls the mass back towards the equilibrium position. As it moves back to this position the mass loses Elastic P.E. but due to the conservation of energy gains Kinetic Energy. When the mass returns to the equilibrium position its Elastic P.E. is zero but all of its initial energy is now in the form of K.E. Consequently the object is moving at maximum speed and cannot remain at the equilibrium position.

The mass moves past the equilibrium position and now once again a restoring force is pulling it back. The mass loses K.E. and gains Elastic P.E. It comes to rest when all of the K.E. has been converted to P.E. and it finds itself at the same distance from the equilibrium as when it started but in the opposite direction.

Once again the mass starts its journey back to the equilibrium position and once again it is unable to remain there due to K.E. As a result the mass is forced to continuously move back and forth, or in other words oscillate.

Every oscillating system has a Time Period, which is the time it takes for the system to completely one cycle, one sequence of motions which is then repeated endlessly. The Time Period, T, of a mass-spring system is given by:

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{6}$$

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Experiment 2: Hooke's Law

Objective

Use springs to investigate Hooke's Law, springs in series and parallel, and oscillations.

Apparatus

- Stand
- Meter Rule
- Two Springs (with different spring constants)
- Slotted Mass Set
- Stopwatch

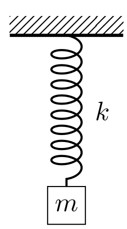
1 Single Spring

Objective

Calculate the spring constant of the two springs using Hooke's Law.

Setup

Choose one of the springs and suspend it from the horizontal arm of the stand. Attach the mass hanger (no additional mass) to the bottom of the spring. This will cause the spring to stretch.



Procedure

Successively add additional mass to the hanger and each time measure the total length l of the spring (once the spring has come to rest). Record you results in a table with the following format. *Note*: m is the total mass you have added to the spring (including that of the hanger).

m (g)	l (cm)

Calculation

Draw a graph of l (y-axis) vs. m (x-axis) including steep and shallow fit lines. Using Hooke's Law $mg = kx$ calculate the the spring constant from the slope of the graph. Calculate the corresponding uncertainty in the value of the spring constant using the steep and shallow fit lines on your graph.

Repeat the procedure and calculations for the second springs.

Record your results in a table with the following format. Pay particular attention to the uncertainty and units of your results.

k_1	
k_2	

2 Springs in Parallel

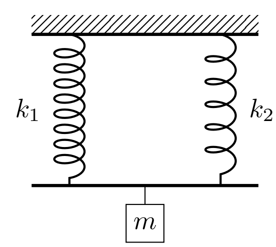
Objective

Verify that two springs in parallel have an effective spring constant given by

$$k = k_1 + k_2 \tag{1}$$

Setup

Using the same two springs from section 1 attach them both to the horizontal arm of the stand, leaving some horizontal space between them. Now place a thin rod through the lower ends of both springs. Attach an empty mass hanger to the center of this thin rod. The mass of the hanger will cause the thin rod to stretch the two springs until they have equal stretched lengths.



Procedure

Follow the procedure from section 1 to create a table and a graph of l vs. m.

Calculations

Use your graph to calculate the effective spring constant $k_{parallel}$ of the two parallel springs. Compare your answer to the expected value from equation (1) by calculating the percentage difference between the two values.

The percentage difference between any two values x_1 and x_2 is defined as

$$\frac{|x_1 - x_2|}{\frac{1}{2}(x_1 + x_2)} \times 100\% \tag{2}$$

3 Springs in Series

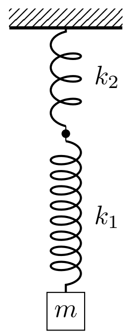
Objective

Verify that two springs in series have an effective springs constant given by

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \tag{3}$$

Setup

Using the same two springs attach one to the horizontal arm on the stand and attach the top of the other spring from the bottom of the first. Now attach the empty mass hanger to the bottom of the lower spring.



Procedure

Follow the procedure from sections 1 and 2 to create a table and a graph of l vs. m.

Calculations

Use your graph to calculate the effective spring constant k_{series} of the two springs in series. Compare your answer the expected value from equation (3) by calculating the percentage difference between the two values (using equation (2)).

4 Oscillating Spring

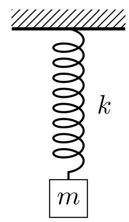
Objective

Calculate the spring constant of a spring using the Time Period of its oscillation as given by

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{4}$$

Setup

Choose a single spring. Attach it to the stand and suspend the mass hanger from its lower end.



Procedure

Successively add additional mass to the hanger. For each new mass added, pull down the mass slightly and release. This will cause the mass to start performing vertical oscillations.

Wait for the mass to oscillate at least twice. The next time the mass passes its equilibrium (middle) position going down start your stopwatch and count zero. The mass will move down, back up, will cross the equilibrium position going up, turn back around and then pass the equilibrium position going down.

Only when it passes the equilibrium position going down does one complete oscillation occur. Only then should you advance your count to one. Keep counting until the mass has completed 20 oscillations at which point stop your stopwatch.

Stop the mass. Repeat the process described above to get two more readings for T_{20}, the time for 20 oscillations.

Record the total mass (including hanger) and T_{20} (the time for 20 oscillations) in a table with the following format. *Only fill in the first four columns which comprise the observations. You can fill out the rest of the columns after you have completed your observations and are starting your calculations.*

m (g)	T_{20_1} (s)	T_{20_2} (s)	T_{20_3} (s)	T (s)	ΔT (s)	T^2 (s^2)	$\Delta(T^2)$ (s^2)

Calculations

Equation (4) is not linear between T and m. To linearize it we square both sides.

$$T^2 = \frac{4\pi^2}{k} m \tag{5}$$

Now the equation is linear between T^2 and m. A graph between these quantities will have a gradient/slope equal to $\frac{4\pi^2}{k}$.

Complete the table above by filling out the remaining columns. T is the time period which is calculating by averaging the T_{20} and dividing by 20 to get the time for a single oscillation.

$$T = \frac{T_{20\_1} + T_{20\_2} + T_{20\_3}}{60} \tag{6}$$

ΔT is the uncertainty associated with the time period T and is calculated using the standard deviation of the three T_{20}.

$$\Delta T = \frac{1}{20} \Delta T_{20} \tag{7}$$

Finally the uncertainty $\Delta(T^2)$ in the squared quantity T^2 is calculated using the propagation of error equation for raising powers.

$$\frac{\Delta(T^2)}{T^2} = 2\frac{\Delta T}{T} \tag{8}$$

Once the table is completed use it to plot a graph of T^2 vs m. Draw a best-fit line as well as steep and shallow lines.

(i) Calculate the value of the slope as well as its associated uncertainty.

(ii) Use the value of the slope and its uncertainty to calculate the value of the spring constant and its associated uncertainty.

(iii) Compare this value to the one calculated in section 1 by calculating the percentage difference (equation (2)) between the values.