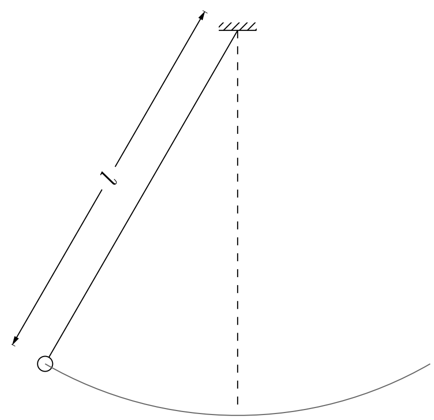
COMSATS Institute of Information Technology,
Islamabad Campus

PHY-108 : **Physics Lab 1** (*Mechanics of Particles*)

Experiment 1: Simple Pendulum

A simple pendulum consists of a small object (known as the bob) suspended from an inextensible string of negligible mass. When left undisturbed the bob hangs motionless with the string vertical. If the bob is pulled to one end and let go it begins to swing back and forth.

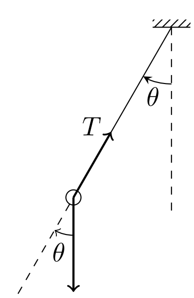
1 Experimental Setup



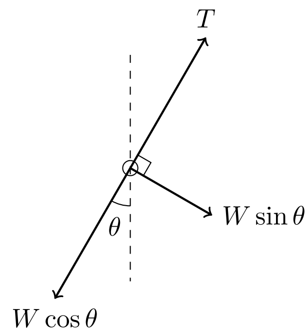
The point about which an object is free to rotate (swing) is known as the pivot (or fulcrum). In the case of the simple pendulum this is the top of the string from which the bob is suspended. The length l of the pendulum is the distance from the pivot to the center of the bob.

2 Free-Body Diagram

(Neglecting air resistance) only two forces act on the bob of a simple pendulum: its own weight and the tension in the string. The weight always acts downwards (since it is caused by the force of gravity of the Earth) while the tension in a string always pulls on an object in the direction of the string itself.



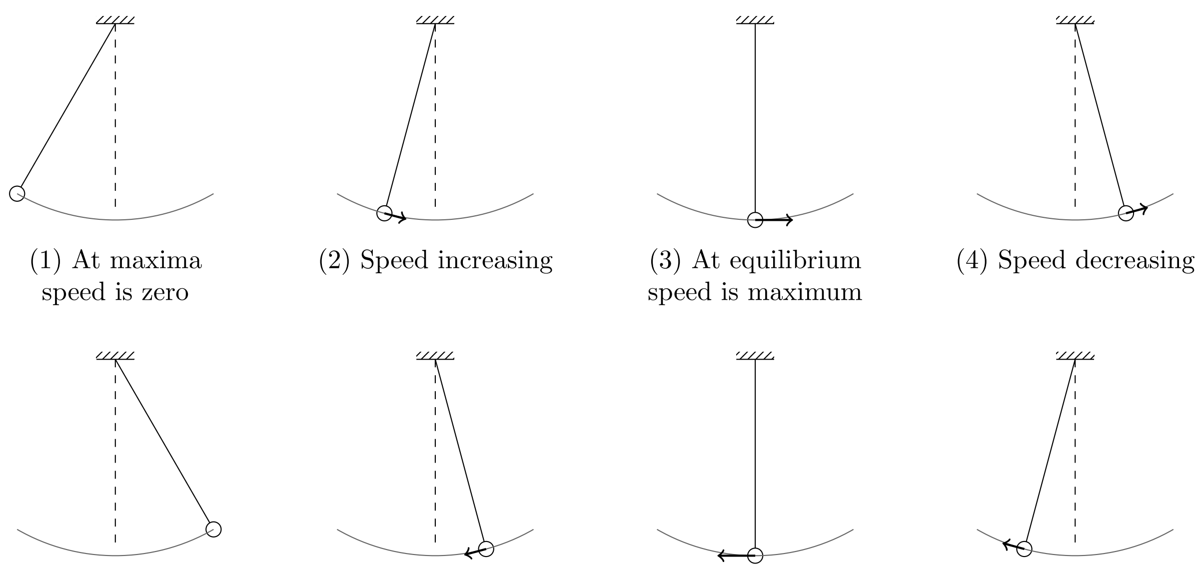
Since for any angle $\theta \neq 0$ the two forces (W and T) are not co-linear (pointing in the same or opposite directions) we need to resolve one of the forces in to its components. We choose to resolve W parallel and perpendicular to the string.



3 Simple Harmonic Motion

Because of the tension in the string the bob cannot move in the radial direction (towards or away from the pivot along the direction of the string). This means that the tension exactly balances $W \cos\theta$ (the component of the weight parallel to the string). This leaves us with the component $W \sin\theta$ unbalanced. Therefore whenever $\theta \neq 0$ there is a net force acting on the bob which is pulling it towards the equilibrium position ($\theta = 0$ where the string is vertical, and T and W exactly balance each other). Since the force is always acting to bring the bob back to the equilibrium position (restore equilibrium) it is known as a **restoring force**.

If the bob is pulled to one side and let go it starts swinging back and forth, in other words it begins to oscillate. Its motion during a single oscillation is described in the figure below.



(1) At maxima speed is zero

(2) Speed increasing

(3) At equilibrium speed is maximum

(4) Speed decreasing

(5) At maxima speed is zero

(6) Speed increasing in opposite direction

(7) At equilibrium again. Maximum speed in opposite direction

(8) Speed decreasing

When $\theta \neq 0$ the unbalanced restoring force $W \sin\theta$ causes the bob to accelerate towards the equilibrium position with its speed increasing. When the bob arrives at the equilibrium position the two forces are in balance and the net acceleration is zero but it is now moving with non-zero (in fact maximum) speed. This means it cannot stay in the equilibrium position and must continue to swing.

As it swings to the other side the unbalanced force $W \sin\theta$ reappears but now it is pointing in the opposite direction (still towards the equilibrium position). This force causes the bob to slow down. Eventually its speed becomes zero and it comes momentarily to rest. This position is known as the maxima (the corresponding angle θ is known as the amplitude of the oscillation).

Since the maxima corresponds to a non-zero angle θ there exists an unbalanced force pulling towards the equilibrium position. This causes the bob to now move back towards the equilibrium position with increasing speed. Once again it arrives at the equilibrium position with non-zero speed and now moving in the opposite direction. Once again it flies past the equilibrium position and its speed is slowed down by $W \sin\theta$ acting towards the equilibrium position. When its speed becomes zero it achieves maxima in the opposite direction and will start to move back.

The bob has now completed one cycle or oscillation. Because of the unbalanced force $W \sin\theta$ and the non-zero speed at the equilibrium position the bob is forced to repeat this motion over and over again. This is a type of simple harmonic motion. The bob is said to be performing oscillations. It can be shown that it takes the bob the same amount of time to complete a single oscillation regardless of the amplitude (how far the maxima is from the equilibrium position - as long as the amplitude is small). This is known as the **Time Period** of the oscillation.

4 Time Period

It can be shown that the time period of a simple pendulum only depends upon the length of the string and strength of the force of gravity.

$$T = 2\pi\sqrt{\frac{l}{g}} \tag{1}$$

This makes pendulums very useful as time-keepers. As long as the length of the string remains fixed the pendulum takes the same amount of time for each oscillation and can be depended upon to keep time.

Experiment 1: Simple Pendulum

1 Objective

Calculate the value of g (acceleration due to gravity) using a simple pendulum.

2 Apparatus

- String
- Mass (Bob)
- Stand
- Alligator Clip
- Stop-watch
- Meter-stick

3 Procedure

A simple pendulum consists of a mass (known as bob) which is attached to a string whose other end is held fixed (the pivot). The bob is then free to swing to and fro about the pivot. The time-period of these oscillations is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad (1)$$

where

- T is the Time Period
- l is the length of the pendulum (from the pivot to the center of the bob)
- g is the acceleration due to gravity

Our aim is to vary the length l of the pendulum, by changing the length of the string, and measuring the corresponding value of the Time Period T. These observations will be used to calculate the value of g.

3.1 Setup

Tie the bob to one end of the string. Make sure that the string is slightly longer than the height of the stand. Wrap the free end of the string around the top of the stand. Use the alligator clip to secure the top end of the string so that the length l from the pivot to the center of the bob does not change as the pendulum oscillates.

3.2 Tabulation

Record your data in a table with the following format.

l (cm)	t_{10_1} (s)	t_{10_2} (s)	t_{10_3} (s)	t_{10_4} (s)	t_{10_5} (s)

For each value of l we will measure the time for 10 oscillations, five times, giving us the five t_{10_i}. This table consists of the measurements we take in the experiment. From these measured values we calculate other derived quantities which will allow us to achieve our objective of calculating the value of g.

It is recommended that you observe and right down all of the measurements first before you calculate the rest of the values. Using a pencil to write down the values will make it easy to fix inevitable mistakes.

3.3 Observation

Start with a length l of 10 cm. Measure and record the five t_{10_i}, that is, measure the time for 10 oscillations five times. Then increase the value of l by 10 cm and measure the five t_{10_i} again.

Keep increasing the value of l by 10 cm until the length of the string exceeds the height of the stand and you are no longer able to swing the pendulum (since the bob starts to touch the table top).

4 Calculations

4.1 Linearizing the Equation

The time period of the oscillations of a simple pendulum is given by equation (1). This equation is not linear in the dependence of T on l since the variable l appears inside a square root. To calculate the value of g we must transform the equation in to a linear form which will allow us to plot a straight-line graph from the values we have measured.

To that end we simply square both sides of (1) giving us

$$T^2 = \frac{4\pi^2}{g} l \tag{2}$$

If we now consider the relationship between T^2 and l, it is linear. Compare this equation to that of the straight line $y = mx + c$ and you can immediately deduce that slope of the T^2 versus l graph will be equal to

$$\text{slope} = \frac{4\pi^2}{g} \tag{3}$$

therefore we will be able to calculate the value of g from the slope of the graph.

4.2 Derived Quantities

Use your measurements (recorded in the table above) to calculate and record the derived quantities in a table with the following format.

l (cm)	$\bar{t}_{10}$ (s)	$\Delta(t_{10})$ (s)	T (s)	$\Delta(T)$ (s)	T^2 (s^2)	$\Delta(T^2)$ (s^2)

where $\bar{t}_{10}$ is the mean/average of the five t_{10_i} given by

$$\bar{t}_{10} = \frac{1}{5}\sum_{i=1}^{5} t_{10_i} = \frac{1}{5}(t_{10_1} + t_{10_2} + t_{10_3} + t_{10_4} + t_{10_5}) \tag{4}$$

$\Delta(t_{10})$ is the standard deviation (uncertainty) in the five values t_{10_i} calculated using

$$\begin{aligned}\Delta(t_{10}) &= \sqrt{\frac{1}{5}\sum_{i=1}^{5}(t_{10_i} - \bar{t}_{10})^2} \\ &= \sqrt{\frac{1}{5}\left[(t_{10_1} - \bar{t}_{10})^2 + (t_{10_2} - \bar{t}_{10})^2 + (t_{10_3} - \bar{t}_{10})^2 + (t_{10_4} - \bar{t}_{10})^2 + (t_{10_5} - \bar{t}_{10})^2\right]}\end{aligned} \tag{5}$$

The time period T (time for one oscillation) is calculated by dividing the mean time for 10 oscillations $\bar{t}_{10}$ by 10, that is

$$T = \frac{\bar{t}_{10}}{10} \tag{6}$$

The corresponding uncertainty ΔT is derived from the equation above and is given by

$$\Delta T = \frac{\Delta(\bar{t}_{10})}{10} \tag{7}$$

Finally, the uncertainty in the derived quantity T^2, denoted by $\Delta(T^2)$, comes from the uncertainty ΔT in T. It is calculated using

$$\frac{\Delta(T^2)}{T^2} = 2\frac{\Delta T}{T} \tag{8}$$

where the factor of 2 comes from T being raised to the power 2 in the expression T^2.

5 Graph

Now that we have values for T^2 versus l (along with the uncertainty $\Delta(T^2)$ it is now time to draw a graph that will allow us to calculate the value of g.

- **Draw a linear graph of T^2 vs. l including error bars.** *Take special care with your choice of scale. Label all axes carefully.*
- **Use your graph to calculate the slope of the best fit line.** *Don't forget to write down the units.*
- **Draw additional steep and shallow fit lines. Use these to calculate and note the uncertainty in the value of the slope.**

6 Results

1. Use the calculated value of the slope, its uncertainty and equation (3) to calculate the value of the acceleration due to gravity g and its associated uncertainty.

$$\frac{\Delta g}{g} = \frac{\Delta(\text{slope})}{\text{slope}} \tag{9}$$

2. The actual value of g is 981 cm/s^2. Compare your calculated value with this by calculating the percentage uncertainty. It is defined as

$$\text{percentage uncertainty} = \frac{|\text{actual value} - \text{measured value}|}{\text{actual value}} \times 100\% \tag{10}$$

3. List the possible sources of error in this experiment.